UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2010**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY of CAMBRIDGE International Examinations

1 Solve the inequality $|2x - 3| > 5$.

2 Show that $\int_0^6 \frac{1}{x+2}\,dx = 2\ln 2$.

3 **(i)** Show that the equation $\tan(x + 45°) = 6\tan x$ can be written in the form

$$6\tan^2 x - 5\tan x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\tan(x + 45°) = 6\tan x$, for $0° < x < 180°$. [3]

4 The polynomial $x^3 + 3x^2 + 4x + 2$ is denoted by $f(x)$.

(i) Find the quotient and remainder when $f(x)$ is divided by $x^2 + x - 1$. [4]

(ii) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. [2]

5 **(i)** Given that $y = 2^x$, show that the equation

$$2^x + 3(2^{-x}) = 4$$

can be written in the form

$$y^2 - 4y + 3 = 0.$$ [3]

(ii) Hence solve the equation

$$2^x + 3(2^{-x}) = 4,$$

giving the values of x correct to 3 significant figures where appropriate. [3]

6 The equation of a curve is

$$x^2y + y^2 = 6x.$$

(i) Show that $\frac{dy}{dx} = \frac{6 - 2xy}{x^2 + 2y}$. [4]

(ii) Find the equation of the tangent to the curve at the point with coordinates (1, 2), giving your answer in the form $ax + by + c = 0$. [3]

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 2 - x$$

has only one root.

(ii) Verify by calculation that this root lies between $x = 0$ and $x = 0.5$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\ln(2 - x_n)$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(i)** By differentiating $\dfrac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) By expressing $\cot^2 x$ in terms of $\operatorname{cosec}^2 x$ and using the result of part **(i)**, show that

$$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx = 1 - \tfrac{1}{4}\pi.$$ [4]

(iii) Express $\cos 2x$ in terms of $\sin^2 x$ and hence show that $\dfrac{1}{1 - \cos 2x}$ can be expressed as $\tfrac{1}{2}\operatorname{cosec}^2 x$. Hence, using the result of part **(i)**, find

$$\int \frac{1}{1 - \cos 2x} \, dx.$$ [3]

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